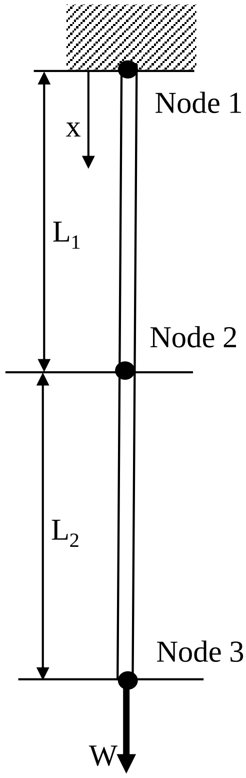
University of Nottingham
Department of Mechanical, Materials and Manufacturing Engineering
MM3CMQ (Computer Modelling Techniques)

SOLUTIONS
Exercise Sheet (FE)-1
(Pin-Jointed Elements)

1 . 1D pin-jointed problem



From Lecture Notes:

$$\begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ W \end{bmatrix} \begin{matrix} \text{— } (1) \\ \text{— } (2) \end{matrix}$$

$$k = \frac{EA_e}{L_e}$$

$$k_1 = \frac{(220\times10^9)(200\times10^{-6})}{2} = 22.0\times10^6 \text{ N/m}$$

$$k_2 = \frac{(90\times10^9)(350\times10^{-6})}{1.5} = 21.0\times10^6 \text{ N/m}$$

$$(1) \rightarrow \quad (k_1+k_2)u_2 - k_2u_3 = 0 \quad \text{— } (3)$$

$$(2) \rightarrow \quad -k_2u_2 + k_2u_3 = W \quad \text{— } (4)$$

$$\text{i.e. } u_3 = u_2 + \frac{W}{k_2}$$

Subs. in (3): $(k_1+k_2)u_2 - k_2\left(u_2 + \frac{W}{k_2}\right) = 0$

$$k_1u_2 + \cancel{k_2u_2} - \cancel{k_2u_2} - W = 0$$

$$u_2 = \frac{W}{k_1}$$

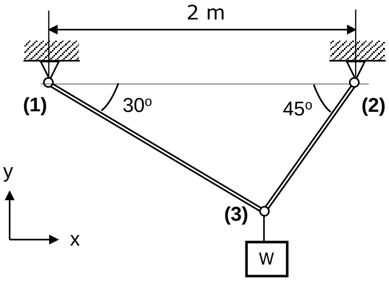
$$\& \quad u_3 = \frac{W}{k_1} + \frac{W}{k_2}$$

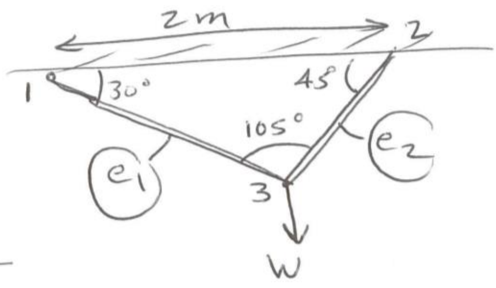
$$u_2 = \frac{50\times10^3}{22\times10^6} = \underline{\underline{2.272\times10^{-3} \text{ m}}}$$

$$u_3 = \frac{50\times10^3}{22\times10^6} + \frac{50\times10^3}{21\times10^6} = (2.272 + 2.381)\times10^{-3}$$

$$= \underline{\underline{4.653\times10^{-3} \text{ m}}}$$

2. 2D Structural analysis problem (2 bars)



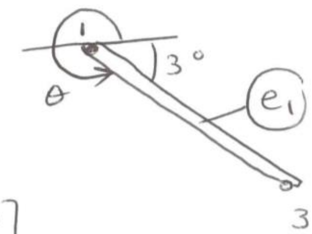


Using sine rule:

$$\frac{2}{\sin 105} = \frac{L_{e_1}}{\sin 45} \Rightarrow L_{e_1} = 1.464\ m$$

$$\frac{2}{\sin 105} = \frac{L_{e_2}}{\sin 30} \Rightarrow L_{e_2} = 1.035\ m$$

Element e_1 (nodes 1–3)



$\theta = 330°$, $L_{e_1} = 1.464\ m$

$\sin\theta = -0.5$

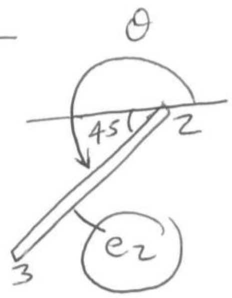
$\cos\theta = 0.866$

$$\frac{AE}{L_{e_1}} = \frac{(250\times10^{-6})(200\times10^{9})}{1.464} = 34.153\times10^{6}$$

$$[K_{e_1}] = \left(\frac{AE}{L_{e_1}}\right)\left[\begin{array}{cc|cc} 0.75 & -0.433 & -0.75 & 0.433 \\ -0.433 & 0.25 & 0.433 & -0.25 \\ \hline -0.75 & 0.433 & 0.75 & -0.433 \\ 0.433 & -0.25 & -0.433 & 0.25 \end{array}\right]$$

$$[K_{e_1}] = (10^{6})\left[\begin{array}{cc|cc} 25.61 & -14.78 & -25.61 & 14.78 \\ -14.78 & 8.54 & 14.78 & -8.54 \\ \hline -25.61 & 14.78 & 25.61 & -14.78 \\ 14.78 & -8.54 & -14.78 & 8.54 \end{array}\right]$$

Element e_2 (nodes 2–3)



$\theta = 225°$, $L_{e_2} = 1.035\ m$

$\sin\theta = -0.7071$

$\cos\theta = -0.7071$

$$\frac{AE}{L_{e_2}} = \frac{(250\times10^{-6})(200\times10^{9})}{1.035} = 48.31\times10^{6}$$

$$[K_{e_2}] = \left(\frac{AE}{L_{e_2}}\right)\left[\begin{array}{cc|cc} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ \hline -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{array}\right]$$

$$[K_{e_2}] = (10^{6})\left[\begin{array}{cc|cc} 24.155 & 24.155 & -24.155 & -24.155 \\ 24.155 & 24.155 & -24.155 & -24.155 \\ \hline -24.155 & -24.155 & 24.155 & 24.155 \\ -24.155 & -24.155 & 24.155 & 24.155 \end{array}\right]$$

Disp. Vector : At nodes 1 and 2 : $u_x = 0,\ u_y = 0$

$$[U]^T = [\ 0 \quad 0 \quad 0 \quad 0 \quad u_{x3} \quad u_{y3}\ ]$$

Force vector At node 3, $F_{x3} = 0$, $F_{y3} = -W$ (downwards)

$$[F]^T = [\ 0 \quad 0 \quad 0 \quad 0 \quad F_{x3} \quad F_{y3}\ ]$$

Assembly

$$\begin{bmatrix} (K_{e1})_{11} & 0 & (K_{e1})_{12} \\ 0 & (K_{e2})_{11} & (K_{e2})_{12} \\ (K_{e1})_{21} & (K_{e2})_{21} & (K_{e1})_{22} + (K_{e2})_{22} \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -W \end{bmatrix}$$

$$(10^6) \begin{bmatrix} 25.61 & -14.78 & & & -25.61 & 14.78 \\ -14.78 & 8.54 & & 0 & 14.78 & -8.54 \\ & & 24.155 & 24.155 & -24.155 & -24.155 \\ & 0 & 24.155 & 24.155 & -24.155 & -24.155 \\ -25.61 & 14.78 & -24.155 & -24.155 & 25.61 + 24.155 & -14.78 + 24.155 \\ 14.78 & -8.54 & -24.155 & -24.155 & -14.78 + 24.155 & 8.54 + 24.155 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -W \end{bmatrix}$$

The system of equations reduces to 2 equations only:

$$(10^6) \begin{bmatrix} 49.75 & 9.365 \\ 9.365 & 32.695 \end{bmatrix} \begin{bmatrix} u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \end{bmatrix} (\times 10^3)$$

$$(49.75) u_{x3} + (9.365) u_{y3} = 0 \quad \text{--- } 1$$

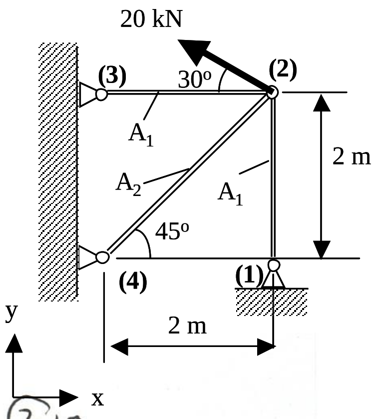
$$(9.368) u_{x3} + (32.695) u_{y3} = -20 \times 10^{-3} \quad \text{--- } 2$$

From the above 2 equations

$$u_{x3} = 0.122 \times 10^{-3} \text{ m}$$

$$u_{y3} = -0.65 \times 10^{-3} \text{ m}$$

3. 2D Structural analysis problem 1 (3 bars)

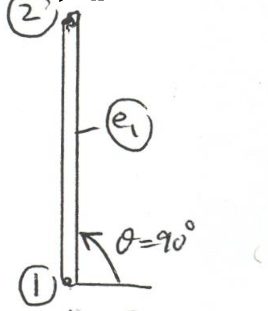


Element (e_1) (nodes 1–2)

$\theta = 90^\circ$, $\sin\theta = 1$, $\cos\theta = 0$

$$\left(\frac{AE}{L}\right)_1 = \frac{250\times10^{-6} \times 200\times10^{9}}{2} = 25\times10^{6}$$

$$[Ke_1] = 25\times10^{6}\left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} (Ke_1)_{11} & (Ke_1)_{12} \\ \hline (Ke_1)_{21} & (Ke_1)_{22} \end{array}\right]$$



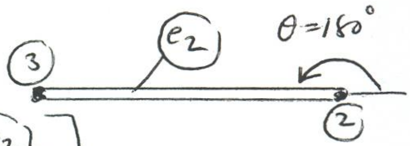
$L_{e_1} = 2m$

$A_{e_1} = A_1 = 250\times10^{-6}\, m^2$

Element (e_2) (nodes 2–3)

$\theta = 180^\circ$, $\sin\theta = 0$, $\cos\theta = -1$

$$\left(\frac{AE}{L}\right)_2 = \frac{250\times10^{-6}\times200\times10^{9}}{2} = 25\times10^{6}$$



$$[Ke_2] = 25\times10^{6}\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} (Ke_2)_{11} & (Ke_2)_{12} \\ \hline (Ke_2)_{21} & (Ke_2)_{22} \end{array}\right]$$

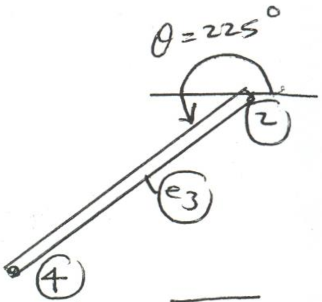
$L_{e_2} = 2m$

$A_{e_2} = A_1 = 250\times10^{-6}\, m^2$

Element (e_3) (nodes 2–4)

$\theta = 225^\circ$, $\sin\theta = -0.7071$, $\cos\theta = -0.7071$

$$\left(\frac{AE}{L}\right)_3 = \frac{400\times10^{-6}\times200\times10^{9}}{2.828} = 28.288\times10^{6}$$



$$[Ke_3] = 28.288\times10^{6}\left[\begin{array}{cc|cc} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ \hline -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{array}\right]$$

$L_{e_3} = \sqrt{2^2+2^2}$

$= 2.8288m$

$A_{e_3} = A_2 = 400\times10^{-6}\, m^2$

$$= \left[\begin{array}{c|c} (Ke_3)_{11} & (Ke_3)_{12} \\ \hline (Ke_3)_{21} & (Ke_3)_{22} \end{array}\right]$$

Displacement vector

At nodes 1, 3 and 4, $u_x = u_y = 0$

$$[u]^T = [\,0\ 0 \mid u_{x2}\ u_{y2} \mid 0\ 0 \mid 0\ 0\,]$$

Force Vector

At node 2 : $F_{x2} = -20\cos30 = -17.32$ KN

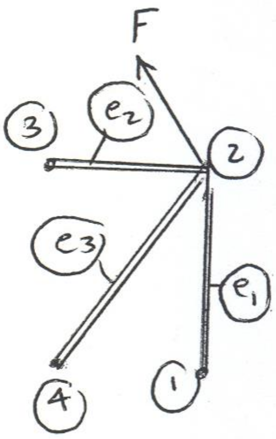
$F_{y2} = +20\sin30 = +10$ KN

$$[F]^T = [\,0\ 0 \mid F_{x2}\ F_{y2} \mid 0\ 0 \mid 0\ 0\,]$$

continued

Matrix Assembly

$$\begin{bmatrix} (ke_1)_{11} & (ke_1)_{12} & 0 & 0 \\ (ke_1)_{21} & (ke_1)_{22} + (ke_2)_{11} + (ke_3)_{11} & (ke_2)_{12} & (ke_3)_{12} \\ 0 & (ke_2)_{21} & (ke_2)_{22} & 0 \\ 0 & (ke_3)_{21} & 0 & (ke_3)_{22} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \hline u_{x2} \\ u_{y2} \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \hline F_{x2} \\ F_{y2} \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \end{bmatrix}$$



Taking only equations (3) and (4) which correspond to node 2:

$$\left[(ke_1)_{22} + (ke_2)_{11} + (ke_3)_{11}\right] \begin{bmatrix} u_{x2} \\ u_{y2} \end{bmatrix} = \begin{bmatrix} F_{x2} \\ F_{y2} \end{bmatrix}$$

$$25\times10^6 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + 25\times10^6 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 28.288\times10^6 \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} -17.32 \\ +10 \end{bmatrix} \times10^3$$

$$\begin{bmatrix} 39.144 & 14.144 \\ 14.144 & 39.144 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{y2} \end{bmatrix} = (10^{-3}) \begin{bmatrix} -17.32 \\ +10 \end{bmatrix}$$

2 equations & 2 unknowns

Eq.(1) : $u_{y2} = (-2.7675)\,u_{x2} - 1.225\times10^{3}$

Subs. in Eq. (2) : $(14.144)\,u_{x2} + (39.144)(-2.7675\,u_{x2} - 1.225\times10^{3}) = 10\times10^{-3}$

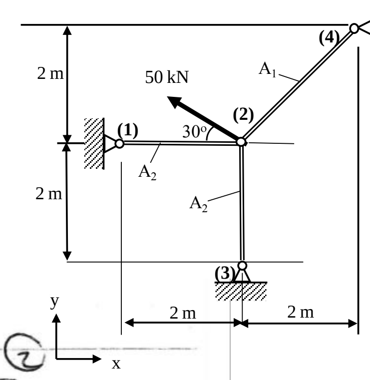
$$(-94.187)\,u_{x2} = 57.951\times10^{-3}$$

$$u_{x2} = -0.615\times10^{-3}\ \text{m} \quad \text{(to the left)}$$

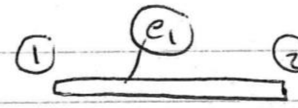
$$\therefore\ u_{y2} = (-2.7675)(-0.615\times10^{-3}) - 1.225\times10^{-3}$$

$$u_{y2} = +0.478\times10^{-3}\ \text{m} \quad \text{(upwards)}$$

4. 2D Structural analysis problem 2 (3 bars)



Element (e_1) : Nodes 1 – 2



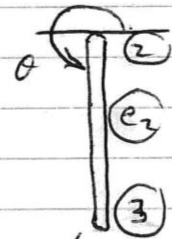
$\theta = 0^\circ$, $\cos\theta = 1$, $\sin\theta = 0$

$$K_{e_1} = \frac{AE}{L_{e_1}} \left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{AE}{L_{e_1}} = \frac{200\times10^{-6}\times 200\times10^{9}}{2} = 20\times10^{6}$$

$$K_{e_1} = (10^6)\left[\begin{array}{cc|cc} 20 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -20 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} (K_{e_1})_{11} & (K_{e_1})_{12} \\ \hline (K_{e_1})_{21} & (K_{e_1})_{22} \end{array}\right]$$

Element (e_2) : Nodes 2 – 3



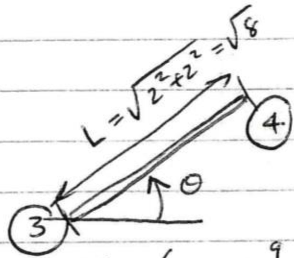
$\theta = 270^\circ$, $\cos\theta = 0$, $\sin\theta = -1$

$$K_{e_2} = \frac{AE}{L_{e_2}} \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right]$$

$$\frac{AE}{L_{e_2}} = \frac{200\times10^{-6}\times 200\times10^{9}}{2} = 20\times10^{6}$$

$$K_{e_2} = (10^6)\left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & -20 \\ \hline 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 20 \end{array}\right] = \left[\begin{array}{c|c} (K_{e_2})_{11} & (K_{e_2})_{12} \\ \hline (K_{e_2})_{21} & (K_{e_2})_{22} \end{array}\right]$$

Element (e_3) : Nodes 2 – 4



$\theta = 45^\circ$, $\cos\theta = 0.707$, $\sin\theta = 0.707 = \frac{1}{\sqrt{2}}$

$$K_{e_3} = \frac{AE}{L_{e_3}} \left[\begin{array}{cc|cc} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ \hline -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{array}\right]$$

$$\frac{AE}{L_{e_3}} = \frac{150\times10^{6}\times 200\times10^{9}}{\sqrt{8}} = 10.607\times10^{6}$$

$$K_{e_3} = (10^6)\left[\begin{array}{cc|cc} 5.303 & 5.303 & -5.303 & -5.303 \\ 5.303 & 5.303 & -5.303 & -5.303 \\ \hline -5.303 & -5.303 & 5.303 & 5.303 \\ -5.303 & -5.303 & 5.303 & 5.303 \end{array}\right] = \left[\begin{array}{c|c} (K_{e_3})_{11} & (K_{e_3})_{12} \\ \hline (K_{e_3})_{21} & (K_{e_3})_{22} \end{array}\right]$$

Displacement vector

At nodes 1, 3, 4 : $u_x = u_y = 0$

$$[U]^T = [\,0 \;\; 0 \mid u_{x2} \;\; u_{y2} \mid 0 \;\; 0 \mid 0 \;\; 0\,]$$

Force Vector

At Node 2: $F_{x_2} = -50\cos 30 = -43.30$ KN

$F_{y_2} = +50\sin 30 = +25.0$ KN

$$[F]^T = [\,0 \;\; 0 \mid F_{x2} \;\; F_{y2} \mid 0 \;\; 0 \mid 0 \;\; 0\,]$$

Matrix Assembly

(b)

$$\begin{bmatrix} (Ke_1)_{11} & (Ke_1)_{12} & 0 & 0 \\ (Ke_1)_{21} & (Ke_1)_{22} + (Ke_2)_{11} + (Ke_3)_{11} & (Ke_2)_{12} & (Ke_3)_{12} \\ 0 & (Ke_2)_{21} & (Ke_2)_{22} & 0 \\ 0 & (Ke_3)_{21} & 0 & (Ke_3)_{21} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x2} \\ u_{y2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_{x2} \\ F_{y2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(10^6)\begin{bmatrix} 20 & 0 & -20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & 0 & (20+0+5.303) & (0+0+5.303) & 0 & 0 & -5.3 & -5.3 \\ 0 & 0 & (0+0+5.303) & (0+20+5.303) & 0 & -20 & -5.3 & -5.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -20 & 0 & 20 & 0 & 0 \\ 0 & 0 & -5.303 & -5.303 & 0 & 0 & 5.3 & 5.3 \\ 0 & 0 & -5.303 & -5.303 & 0 & 0 & 5.3 & 5.3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x2} \\ u_{y2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -43.3 \\ +25.0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} (10^3)$$

(c) The system of equations can be reduced to two equations only

$$\begin{bmatrix} 25.303 & 5.303 \\ 5.303 & 25.303 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{y2} \end{bmatrix} = \begin{bmatrix} -43.3 \\ 25.0 \end{bmatrix} \times 10^{-3}$$

$$(25.303)\, u_{x2} + (5.303)\, u_{y2} = -43.3 \times 10^{-3} \quad \text{--- (1)}$$

$(5.303)\, u_{x2} + (25.303)\, u_{y2} = +$ [illegible] $\times 10^{-3}$ --- (2)

From (1): $u_{x2} = -1.711 \times 10^{-3} - (0.2095)\, u_{y2}$

Subs. in (2): $(5.303)\left[-1.711 \times 10^{-3} - (0.2095)\, u_{y2}\right] + 25.303\, u_{y2} = 25 \times 10^{-3}$

$$(24.192)\, u_{y2} = +34.073 \times 10^{-3}$$

$$u_{y2} = +1.408 \times 10^{-3} \text{ m}$$

$$u_{x2} = -2.006 \times 10^{-3} \text{ m}$$

(d) Stress in element 2-4

$$[\sigma_e] = \frac{E}{L_e}\left[-\cos\theta \;\; -\sin\theta \;\; \cos\theta \;\; \sin\theta\right] \begin{bmatrix} u_{x2} \\ u_{y2} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

$$= \frac{200 \times 10^9}{\sqrt{8}} \left((-\cos 45)\, u_{x2} - \sin\theta\, (u_{y2}) + 0 + 0 \right)$$

$$= 70.7 \times 10^9 \left((-0.707)(-2.006 \times 10^{-3}) - (0.707)(1.408 \times 10^{-3}) \right)$$

$= 29.89 \times 10^6$ Pa (tensile)

5. Torsional Element

This question demonstrates that the principle of minimum TPE can be applied to other physical problems. Here the variables on the torsional element are the angle (θ) and the torque (T), whereas in the lecture notes we used "displacement" and "force" as the variables. The same derivation steps based on the principle of minimum TPE can also be used in the torsional element.

(a) Mini. Total Potential Energy:

Given: $T.P.E. = \int_0^L \frac{GJ}{2}\left(\frac{d\theta}{dx}\right)^2 . dx - T_1\theta_1 - T_2\theta_2$

Assuming a linear variation of θ along the length L:

$$\boxed{\theta = c_1 x + c_2}$$

at $x=0$, $\theta = \theta_1 \Rightarrow \theta_1 = c_1(0) + c_2 \Rightarrow c_2 = \theta_1$

at $x = L$, $\theta = \theta_2 \Rightarrow \theta_2 = c_1(L) + \theta_1 \Rightarrow c_1 = \frac{\theta_2 - \theta_1}{L}$

$$\frac{d\theta}{dx} = c_1 = \frac{\theta_2 - \theta_1}{L}$$

Subs. in T.P.E. $\Rightarrow TPE = \int_0^L \frac{GJ}{2}\left(\frac{\theta_2 - \theta_1}{L}\right)^2 dx - T_1\theta_1 - T_2\theta_2$

$$TPE = \frac{GJ}{2}\frac{(\theta_2-\theta_1)^2}{L^2} \times L - T_1\theta_1 - T_2\theta_2$$

$$= \frac{1}{2}\frac{GJ}{L}\left(\theta_2^2 - 2\theta_1\theta_2 + \theta_1^2\right) - T_1\theta_1 - T_2\theta_2$$

or, in matrix form: $TPE = \left(\frac{1}{2}\frac{GJ}{L}\right)[\theta_1 \;\; \theta_2]\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} - [\theta_1 \; \theta_2]\begin{bmatrix}T_1\\ T_2\end{bmatrix}$

Apply Principle of Min. T.P.E., i.e. $\frac{\partial TPE}{\partial[\theta]} = 0$

$$\frac{\partial TPE}{\partial \theta_1} = 0 = \frac{1}{2}\frac{GJ}{L}(-2\theta_2 + 2\theta_1) - T_1 - 0$$

$$\Rightarrow \boxed{T_1 = \frac{GJ}{L}(\theta_1 - \theta_2)}$$

$$\frac{\partial TPE}{\partial \theta_2} = 0 = \frac{1}{2}\frac{GJ}{L}(2\theta_2 - 2\theta_1) - 0 - T_2$$

$$\Rightarrow \boxed{T_2 = \frac{GJ}{L}(\theta_2 - \theta_1)}$$

In matrix form: $\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \left(\frac{GJ}{L}\right)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$

(b) Torsion Equation

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow T = GJ\left(\frac{\theta}{L}\right)$$

Hence $T_1 = \frac{GJ}{L}(\theta_1 - \theta_2)$

$T_2 = \frac{GJ}{L}(\theta_2 - \theta_1)$

or $\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \left(\frac{GJ}{L}\right)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$ ... as (a)